Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 70138

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2022.

Third Semester

Mechanical Engineering

MA 3351 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to Aeronautical Engineering/Aerospace Engineering/ Automobile Engineering/Biomedical Engineering/ Civil Engineering/Manufacturing Engineering/Marine Engineering/ Materials Science and Engineering/Mechanical Engineering (Sandwich)/ Mechanical and Automation Engineering/Mechatronics Engineering/ Medical Electronics/ Petrochemical Engineering/Production Engineering/ Robotics and Automation/Safety and Fire Engineering/Bio Technology/ Biotechnology and Biochemical Engineering/Food Technology/ Petrochemical Technology/ Petroleum Engineering/Pharmaceutical Technology)

(Regulations 2021)

Time : Three hours　　　　Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the complete solution of the PDE $p^2 + q^2 = 4pq$.

2. Find the complementary function of the PDE $(D^3 - 3DD'^2 + 2D'^3)z = e^{2x-y}$.

3. Identify the given $f(x)$ is an even or odd function. Also sketch its graph.

$$f(x) = \begin{cases} \pi + x & -\pi \le x \le -\dfrac{\pi}{2} \\ -x & -\dfrac{\pi}{2} \le x \le 0 \\ x & 0 \le x \le \dfrac{\pi}{2} \\ \pi - x & \dfrac{\pi}{2} \le x \le \pi \end{cases}$$

4. State Parseval's identity in Fourier series.

5. Write down the appropriate solution of the one dimensional heat flow equation. How is it chosen?

6. The ends A and B of a rod 30 cm long, have their temperature kept at 10°C and 100°C respectively. Then obtain the steady state temperature.

7. What are the sufficient conditions for the existence of Fourier transform of a function $f(x)$?

8. Obtain the Fourier cosine transform of $\frac{1}{2^t}$.

9. Find the inverse Z transform of $\frac{z}{(z-1)^2}$.

10. State final value theorem in Z transform.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Solve : $(x-2z)p+(2z-y)q = y-x$. (8)

(ii) Solve : $\left(D^3 + D^2D' - 4DD'^2 - 4D'^3\right)z = \cos(2x+y)$. (8)

Or

(b) (i) Solve the PDE $2z + p^2 + qy + 2y^2 = 0$. (8)

(ii) Solve $\left(D^2 + 2DD' + D'^2 - 2D - 2D'\right)z = \sin(x+2y)$. (8)

12. (a) (i) Obtain the Fourier series of periodicity 2π for $f(x) = e^x$ in the interval $0 < x < 2\pi$. (8)

(ii) Obtain the half range Fourier cosine series of $f(x) = x(l-x)$ in $(0, l)$. (8)

Or

(b) The following table gives the variations of periodic current over a period.

t sec :	0	T/6	T/3	T/2	2T/3	5T/6	T
A amp :	1.98	1.30	1.05	1.30	–0.88	–0.25	1.98

Show that there is a direct current part of 0.75 amp in the variable current and obtain the amplitude of the first harmonic. (Harmonic Analysis). (16)

13. (a) A tightly stretched string of length $2l$ is fastened at both ends. The midpoint of the string is displaced by a distance 'b' transversely and the string is released from rest in this position. Find the displacement y at any distance x from one end at any time t. (16)

Or

(b) An infinitely long metal plate in the form of an area is enclosed between the lines $y=0$ and $y=\pi$ for positive values of x. The temperature is zero along the edges $y=0$ and $y=\pi$ and the edge at infinity. If the edge $x=0$ is kept at temperature 'ky', find the steady state temperature at any point in the plate. (16)

14. (a) (i) Find the Fourier transform of $f(x)$ given by $f(x)=\begin{cases}1, & for\ |x|\le a\\ 0, & for\ |x|>a\end{cases}$. (8)

(ii) Find the Fourier sine transform of $f(x)=e^{-ax}$, $a>0$ and hence find $F_C\left(xe^{-ax}\right)$. (8)

Or

(b) (i) Using Parseval's identity for Fourier transforms, evaluate $\int_0^\infty \frac{ds}{\left(a^2+s^2\right)\left(b^2+s^2\right)}$. (8)

(ii) Find the Fourier cosine transform of $f(x)=\begin{cases}x & ;\ 0<x<1\\ 2-x & ;\ 1<x<2\\ 0 & ;\ x>2\end{cases}$. (8)

15. (a) Using Z transform, solve the difference equation $u_{n+2}+4u_{n+1}+3u_n=3^n$ with $u_0=0$, $u_1=1$. (16)

Or

(b) State and prove convolution theorem in Z transforms and use it to find $Z^{-1}\left\{\frac{z^2}{(z-a)(z-b)}\right\}$. (16)
